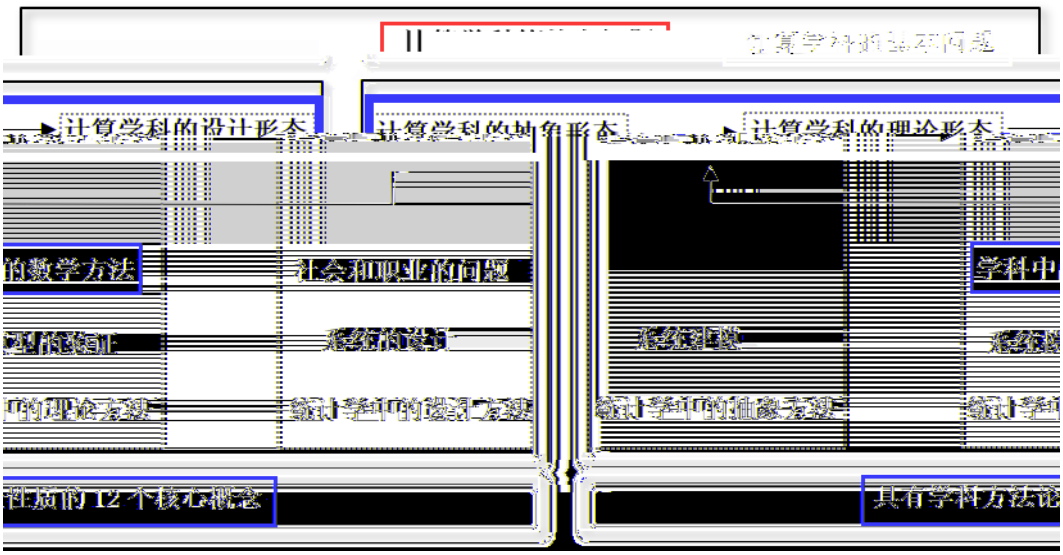
# 科学思维-样例：RSA 公开密钥密码系统

“元认知知识”

“创造”

1.

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1976 Whitfield Diffie

Martin Hellman Diffie – Hellman key exchange DH

*New Directions in Cryptography*

1978 R. L. Rivest A. Shamir

L. M. Adleman *A Method for Obtaining Digital Signatures and Public-Key Cryptosystems*

RSA RSA “ ”

RSA

2002

RSA=<*p*, *q*, *n*, *m*, *e*, *d*, *k*, *c*>

1 *p*, *q*, *n*, *m*, *e*, *d*, *k*, *c* $Z^*$, $Z^*$={1,2,3,…}

2 *p*, *q* $n = p \times q$

3 (*e*, *n*): (*d*, *n*):

4 *m*: $m < n$

5 *c*:

6 $k(m^{k(p-1)(q-1)} (\text{mod } n)) = 1$

7 $k(\ ed = k(p-1)(q-1)+1)$

8 $c = m^e (\text{mod } n)$

9 $m = c^d (\text{mod } n)$

1 *p*, *q*

2 *e* *e* $(p-1)(q-1)$ $0<e<(p-1)(q-1)$

3 *d* $k(\ ed = k(p-1)(q-1)+1)$

3. RSA

1 *m* $c = m^e (\text{mod } n)$

2 *c* $m = c^d (\text{mod } n)$

RSA (*e*, *n*) (*d*, *n*) *p* *q*

$k(m^{k(p-1)(q-1)} (\text{mod } n)) = 1$ CS

例 *p*=3, *q*=11, $n = 3 \times 11 = 33$

*m*=2 $m<n$ , *k*=1

$m^{k(p-1)(q-1)} (\text{mod } n) = 2^{1 \times (3-1) \times (11-1)} (\text{mod } 33)$

$= 2^{20} (\text{mod } 33)$

$= 1\ 048\ 576\ (\text{mod } 33)$

$= 1$

$m=2 \quad m<n \quad , k=2$

$m^{k(p-1)(q-1)} \pmod{n} = 2^{2\times(3-1)\times(11-1)} \pmod{33}$

$= 2^{40} \pmod{33}$

$= 1 \text{ 099 511/627 776} \pmod{33}$

$= 1$

$m=2 \quad m<n \quad , k=3$

$m^{k(p-1)(q-1)} \pmod{n} = 2^{3\times(3-1)\times(11-1)} (\text{mod}$

=2187 (mod 33)

=9

例 $p$=223092827, $q$=218610473 $n$=487 704 284 333 771 171 RSA

$p$, $q$, $n$ “ ”

***e***

$p$=223 092 827, $q$=218 610 473 $(p-1)\times(q-1)$ = (223 092 827–1)×(218 610 473–1)

= 48 770 427 991 673 872

RSA $e$ 48 770 427 991 673 872

$e$=2 48 770 427 991 673 872 mod 2 = 0

$e$=3 48 770 427 991 673 872 mod 3 = 1

3 48 770 427 991 673 872 $e$=3

***d***

$k$ $ed = k(p-1)(q-1)+1$ $k$

$d = (k(p-1)(q-1)+1)/e$

$e$ = 3 $p$=223 092 827, $q$=218 610 473

$d$ =(48 770 427 991 673 872k+1)/3

$k$=1 $d$=48 770 427 991 673 873/3

$k$=2 d=97 540 855 983 347 745/3

=32 513 618 661 115 915

⅝ $d$ $d$=32 513 618 661 115 915 Ա u ضذ

RSA (3, 487 704 284 333 771 171)

(32 513 618 661 115 915, 487 704 284 333 771 171)

RSA $e, n$ ($c$)

$p$ $q$ $n=p\times q$ , ض × ŏ “ ” ”

p=11, q=13 RSA 9

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